

OSA Advanced Photonics Congress 2021

26 July 2021 – 30 July 2021

OSA Virtual Event

Nonlinear Photonic Resonators With Graphene: Saturable Absorption and the Effect of Carrier Diffusion and Finite Relaxation Time

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Introduction

□ Motivation

- Graphene is the most well-studied 2D material with attractive linear and nonlinear properties.
- Graphene **Saturable Absorption (SA)** is one of the most prominent nonlinear effects.
- Exploit graphene SA in integrated nanophotonic **routing elements** and **mode-locked lasers** (MLLs).
- More accurate modeling of nonlinear response of graphene.
- Investigate the interplay amongst various physical effects in the overall nonlinear response.

□ Main Objectives

- Model SA effect by carefully incorporating:
 - **finite relaxation time**
 - **carrier diffusion**
- Apply the model in a practical 3D **graphene-enhanced silicon-on-insulator (SOI)** resonator.

Presentation Outline

□ Theoretical Framework

- SA Model
- Rate Equation (finite relaxation time + carrier diffusion)
- Perturbation Theory and Temporal Coupled Mode Theory (CMT)

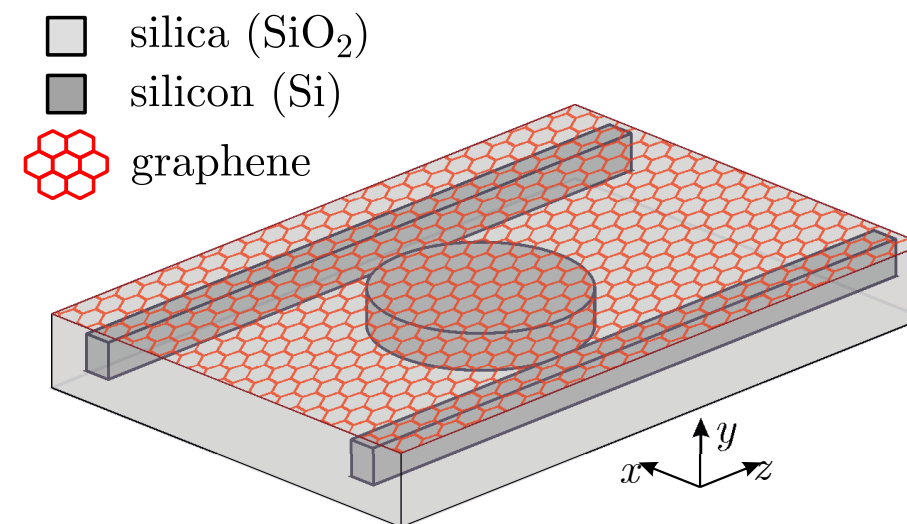
□ Graphene-enhanced Silicon Disk Resonator

- Practical Physical Structure
- Effect of Finite Relaxation Time
- Effect of Carrier Diffusion
- Interplay between SA and the Kerr effect

□ To probe further

- Graphene Hot Electron Model (GHEM)

□ Summary and Conclusions





Theoretical Framework

Theoretical Framework / SA Model, Rate Equation

□ Graphene surface conductivity

$$\sigma_1 = \underbrace{\sigma_{\text{intra}}}_{\substack{\text{Nonsaturable} \\ \text{intraband} \\ \text{component}}} + \underbrace{\sigma_{\text{inter}}(N_c)}_{\substack{\text{Saturable} \\ \text{interband} \\ \text{component}}}$$

- σ_1 depends on carrier density N_c

□ Saturable absorption model

$$\sigma_{\text{inter}}(N_c) = \sigma_0 \left(1 - \frac{N_c}{2N_{\text{sat}}} \right)$$

- N_{sat} : Phenomenological carrier saturation density.
- $\sigma_0 = e^2/(4\hbar) \cong 61 \mu\text{S}$
- Changes to the imaginary part of σ_{inter} are also anticipated.

□ Carrier Rate Equation

$$\frac{dN_c}{dt} = \underbrace{\frac{\frac{1}{2}\text{Re}\{\sigma_{\text{inter}}(N_c)\}|\mathbf{E}_{||}|^2}{\hbar\omega}}_{\text{Source term}} - \underbrace{\frac{N_c}{\tau}}_{\substack{\text{Finite relaxation} \\ \text{time term}}} + \underbrace{D\nabla^2 N_c}_{\text{Diffusion term}}$$

Chatzidimitriou, **Phys Rev A**, **102**, 053512, 2020

□ Saturation Intensity

- τ and N_{sat} are **intrinsic** properties of graphene.
- $I_{\text{sat}} = I_{\text{sat}}(\tau, N_{\text{sat}})$ as:

$$I_{\text{sat}} = \frac{2\hbar\omega N_{\text{sat}}}{\sigma_0\tau Z_0} \propto \tau^{-1}$$

Marini, **Phys. Rev. B** **95**, 125408, 2017

Theoretical Framework / Perturbation Theory and CMT

□ Rigorous framework based on perturbation theory and temporal Coupled Mode Theory (CMT) to study graphene-enhanced resonant configurations.

○ Complex frequency shift :
$$\Delta\tilde{\omega} = \frac{j \iint_S \sigma_1(N_c) |\mathbf{E}_{0,\parallel}|^2 dS}{\iiint_V \epsilon_0 \frac{\partial[\omega\epsilon_r(\omega)]}{\partial\omega} \Big|_{\omega=\omega_0} |\mathbf{E}_0|^2 dV + \iiint_V \mu_0 |\mathbf{H}_0|^2 dV}$$

The entire presence of graphene is treated as perturbation in the NIR !

Christopoulos, *J. Appl. Phys.*, **127**, 223102, 2020

○ CMT equations for a travelling-wave resonator side-coupled to a waveguide:

$$\frac{da(t)}{dt} = j\omega_0 a(t) + j \underbrace{\Delta\omega_{\text{Kerr}}}_{\text{Kerr effect}} + j \underbrace{\Delta\omega_{\text{intra}}}_{\text{power-independent}} a(t) - \left[\underbrace{\frac{1}{\tau_{\text{intra}}}}_{\text{power-independent}} + \frac{1}{\tau_{\text{rad}}} + \underbrace{\frac{1}{\tau_{\text{inter}}(N_c)}}_{\text{power-dependent}} + \frac{1}{\tau_e} \right] a(t) + j \sqrt{\frac{2}{\tau_e}} s_{\text{in}}(t)$$

$$s_{\text{out}}(t) = s_{\text{in}}(t) + j \sqrt{\frac{2}{\tau_e}} a(t)$$

Carrier-diffusion problem is solved concurrently with the CMT cavity-amplitude equation

$$\frac{dN_c}{dt} = \frac{\frac{1}{2} \text{Re}\{\sigma_{\text{inter}}(N_c)\} |\mathbf{E}_{\parallel}|^2}{\hbar\omega} - \frac{N_c}{\tau} + D\nabla^2 N_c$$

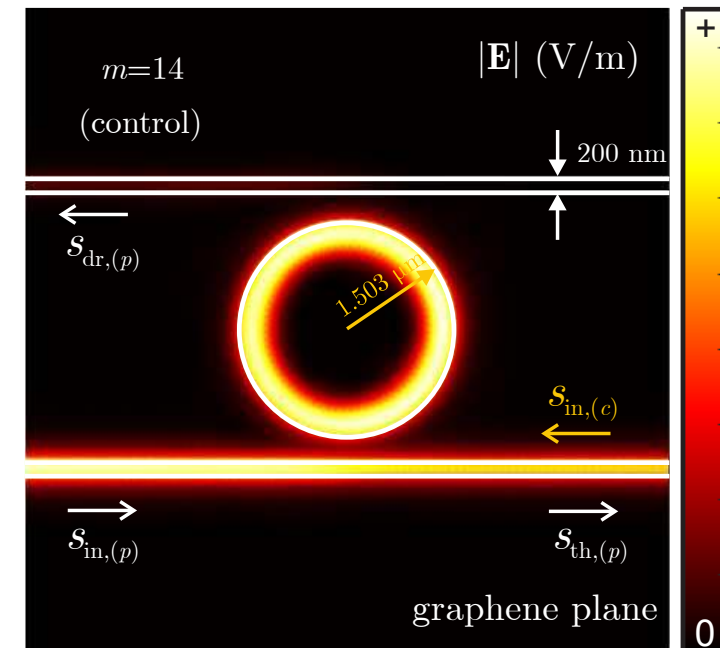
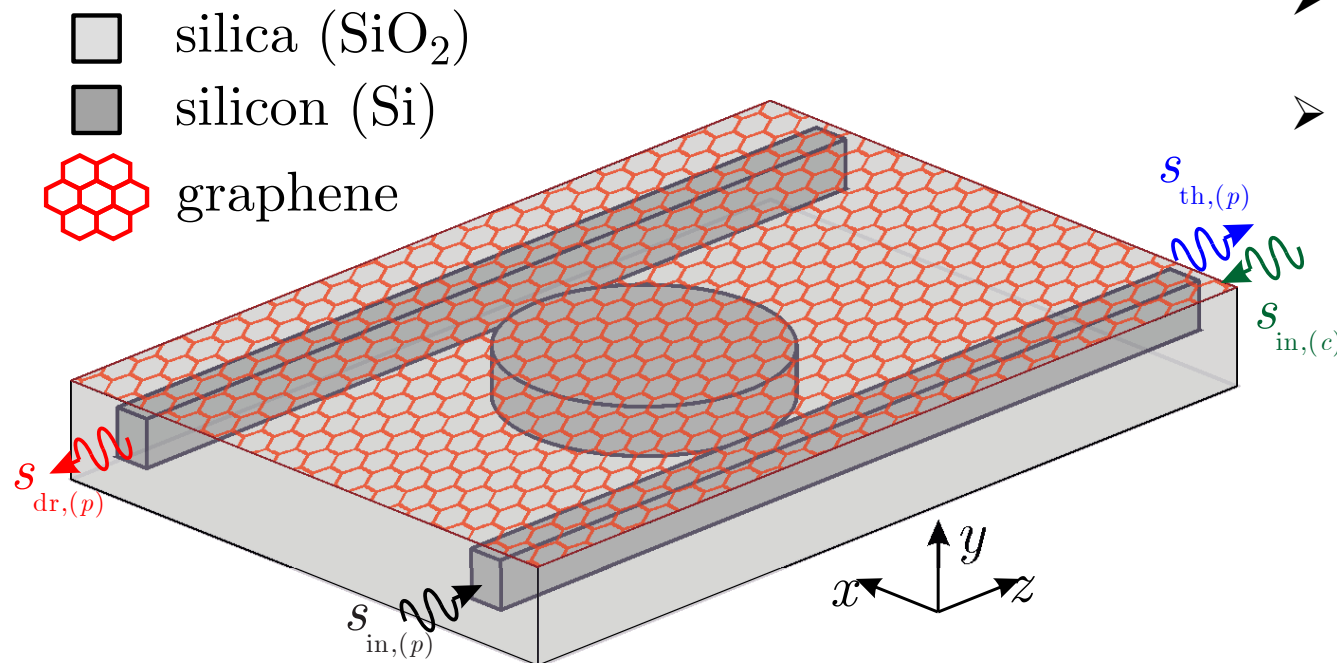
Spatial dependence of N_c fully retained!



Graphene-enhanced Silicon Disk Resonator

Graphene-enhanced Silicon Disk Resonator / Physical Structure

- SOI disk resonator covered with a graphene monoatomic layer and side-coupled to two bus waveguides in an add-drop configuration.
- Two-wave excitation scheme. The output port of the probe (weak) wave is determined by the control (strong) wave.
 - In the **absence** of the control wave, the probe is transmitted to the through port.
 - For appropriate control power the **losses are quenched** and the probe wave is directed to drop port.
 - Power-dependent routing is achieved.



Graphene-enhanced Silicon Disk Resonator / Finite Relaxation Time

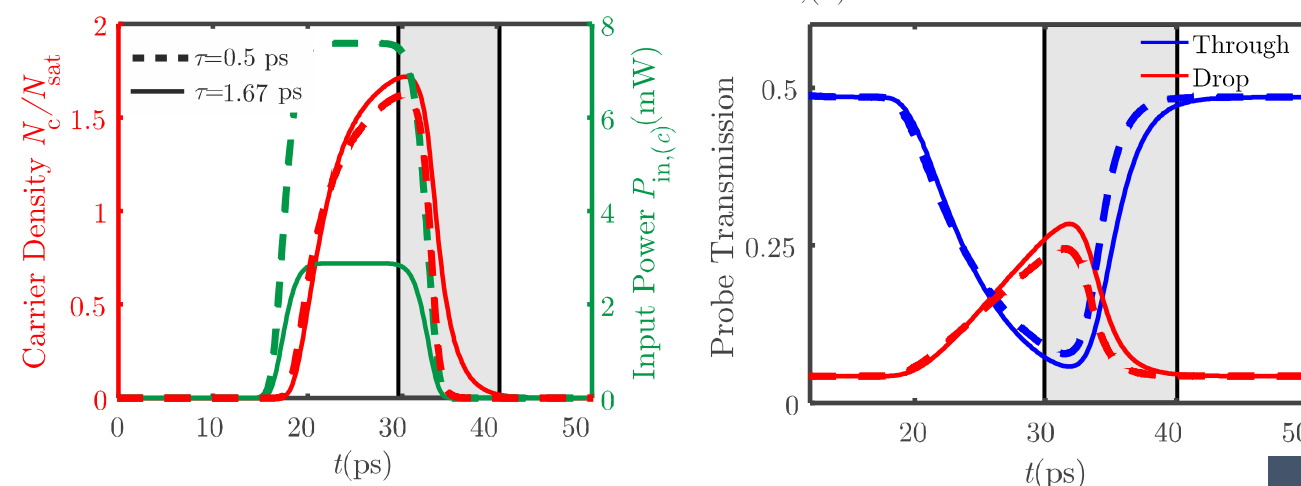
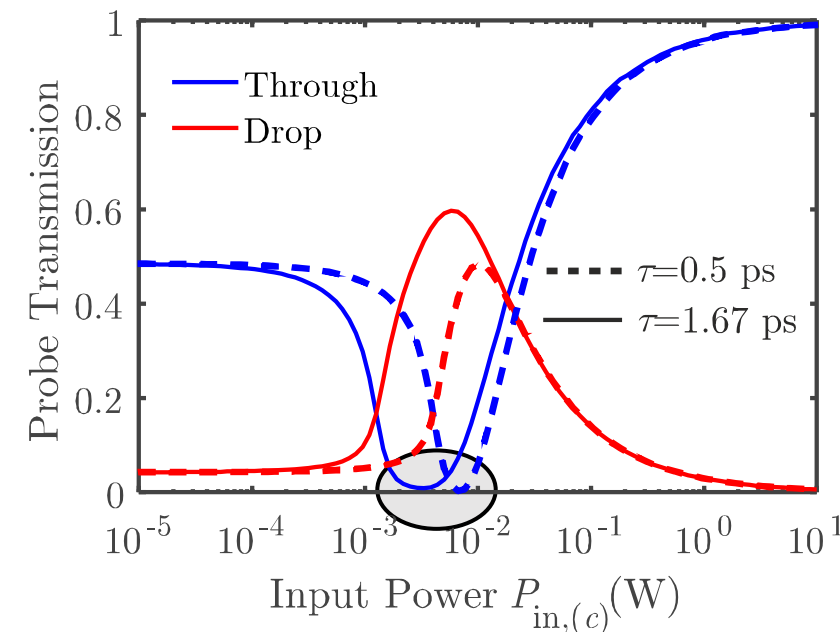
- Study the effects of finite relaxation time and carrier diffusion **separately**.

$$\frac{dN_c}{dt} = \frac{\frac{1}{2}\text{Re}\{\sigma_{\text{inter}}\}|\mathbf{E}_{||}|^2}{\hbar\omega} - \frac{N_c}{\tau} + \cancel{D\nabla^2 N_c}$$

$$\begin{aligned} \tau = 1.67 \text{ ps} &\rightarrow I_{\text{sat}} = 1 \text{ MW/cm}^2 \\ \tau = 0.5 \text{ ps} &\rightarrow I_{\text{sat}} = 3.35 \text{ MW/cm}^2 \end{aligned}$$

$$N_{\text{sat}} = 1.5 \times 10^{15} \text{ m}^{-2}$$

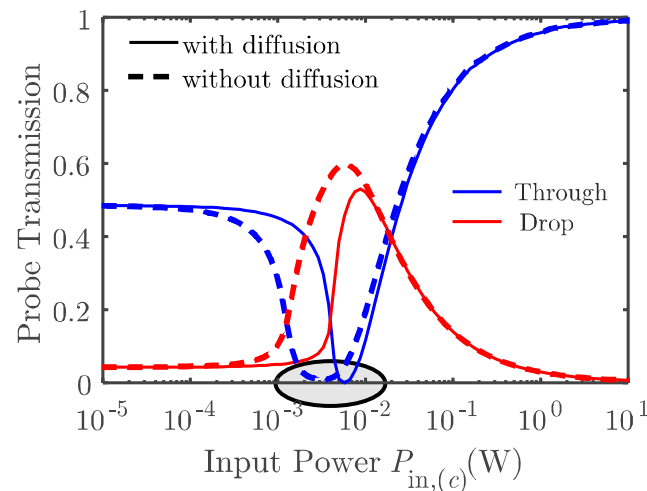
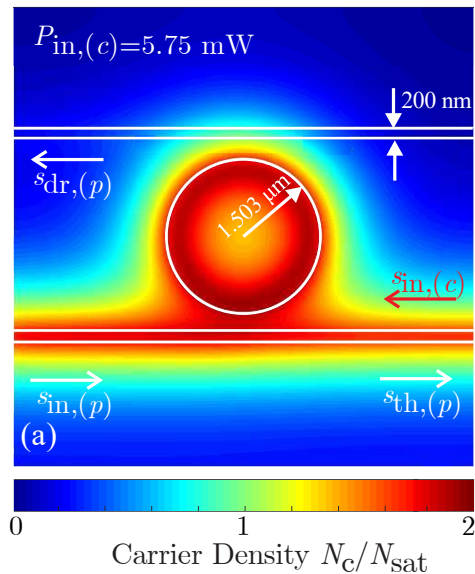
- Low SA relaxation time \implies High saturation intensity \implies High input power to meet **critical coupling condition** (zero through-port transmission).
- Carriers are generated **instantaneously** \implies Delay in the **leading edges** of the transmitted pulses due to **cavity photon-lifetime** ($\tau_{\ell}^{(0)} = 1.27 \text{ ps}$).
- Control is **switched-off** \implies Delay in the **trailing edges** of the transmitted pulses due to **finite relaxation time** (carrier recombination time).
- Opting for faster resonant element/graphene response results in higher power requirements.



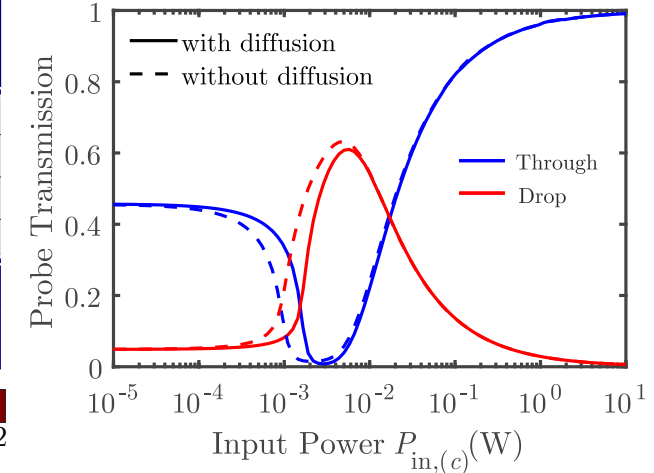
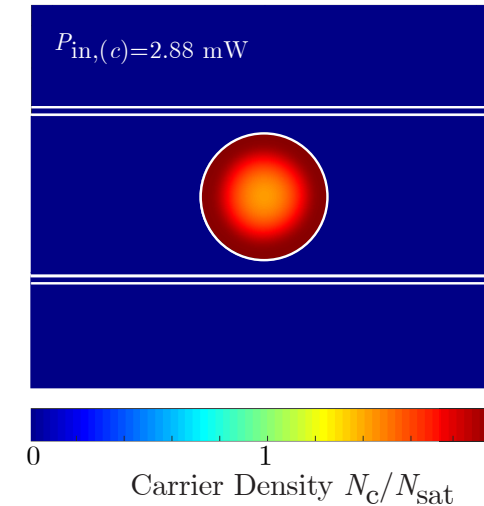
Graphene-enhanced Silicon Disk Resonator / Carrier Diffusion (1/2)

- Diffusion coefficient: $D = 5500 \text{ cm}^2/\text{s}$.
- Diffusion results in **higher input power** requirement to meet the critical coupling condition and **steeper changes** in the transmission curves.
- By limiting the extent of graphene to the place where the mode is confined, **carriers cannot diffuse** due to the zero-outward current (**Neumann B/C**).

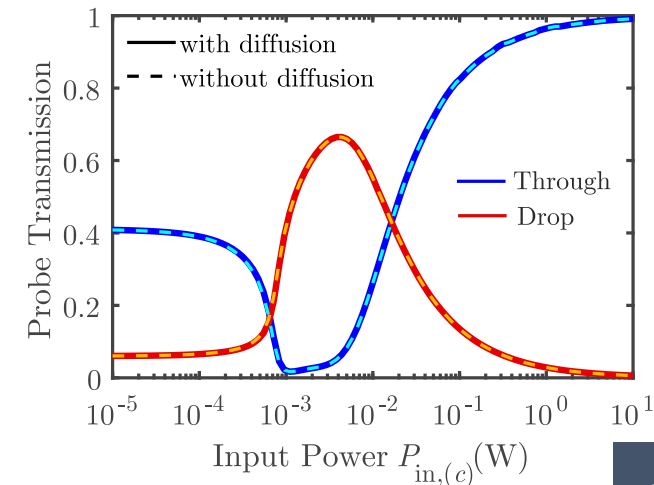
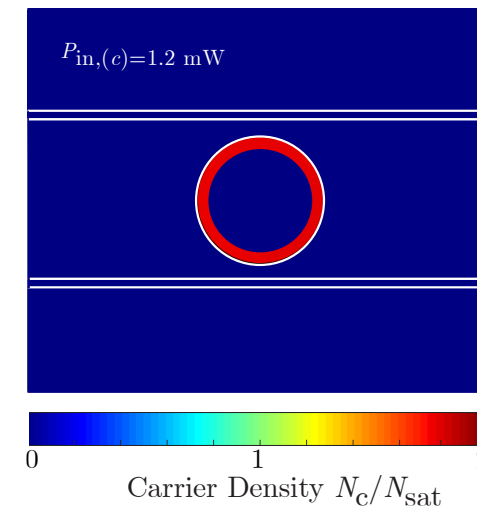
A. Graphene occupies the entire domain.



B. Graphene is a disk matching the resonator.



C. Graphene is a stripe along the resonator circumference.



Graphene-enhanced Silicon Disk Resonator / Carrier Diffusion (2/2)

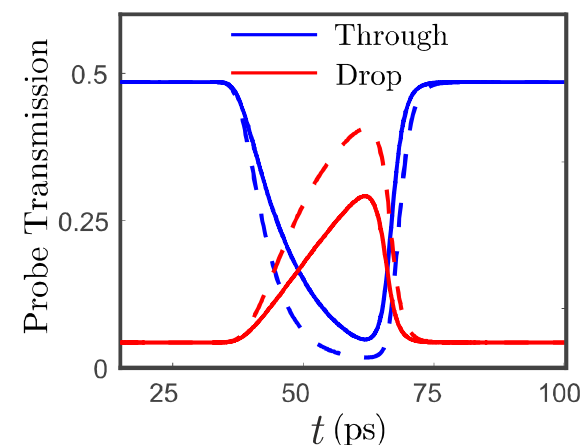
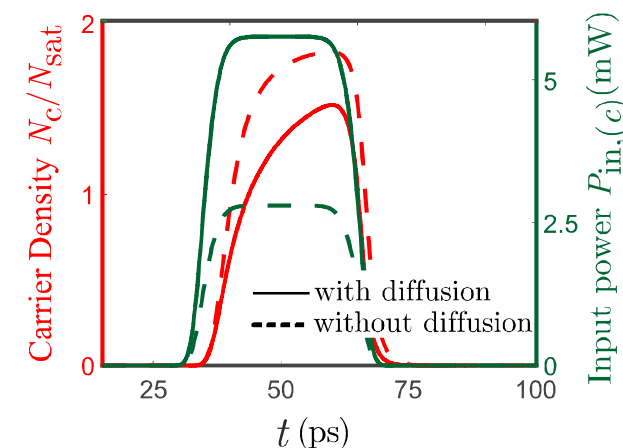
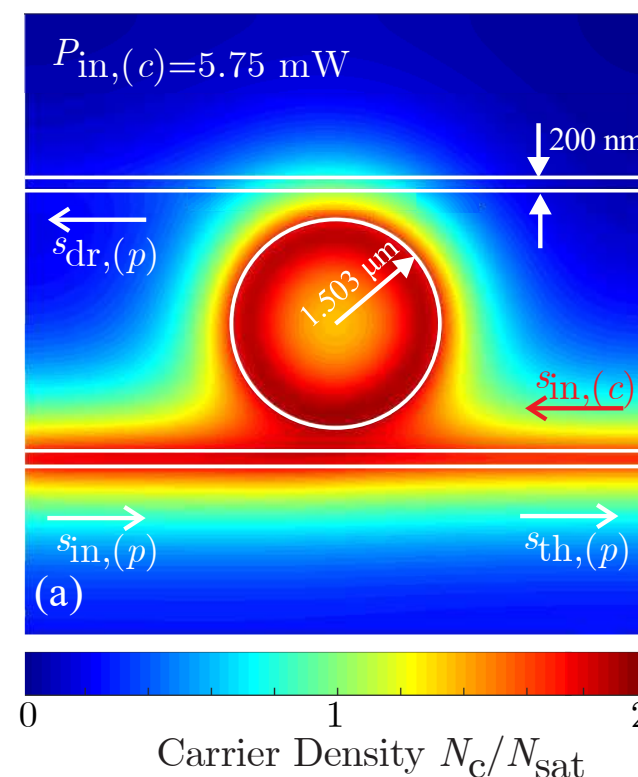
- Graphene occupies the entire domain.

$$\tau = 1.67 \text{ ps} \rightarrow I_{\text{sat}} = 1 \text{ MW/cm}^2$$

$$Q_{\ell}^{(0)} = 773 \rightarrow \tau_{\ell}^{(0)} = 1.27 \text{ ps}$$

Similar contribution
to the temporal
response of the
resonator

- Control input pulse with duration of 30 ps (FWHM), while the **probe input power** is **low** and constant (1 μW).
- Diffusion** results in more pronounced **delay** and **distortion** of the transmitted pulses.
- Both finite relaxation time and carrier diffusion should be considered for a more **precise evaluation** of the nonlinear **resonator's speed** in optical communication applications (e.g. switching and routing elements).



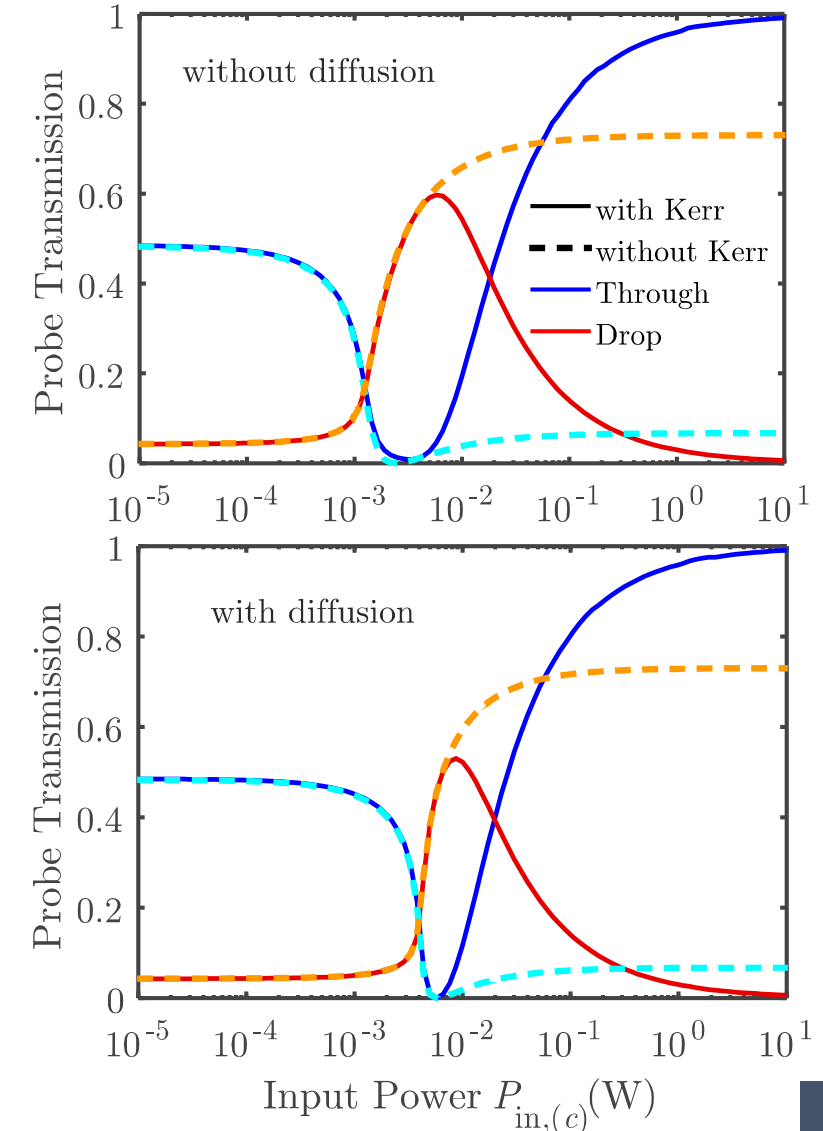
Graphene-enhanced silicon disk resonator / SA & Kerr Interplay

- Nonlinear phase effects are crucial to the overall nonlinear response of resonators.
- Graphene exhibits strong Kerr effect**, which dominates over silicon's.

$$\sigma_{3,\text{gr}} = -j1.2 \times 10^{-20} \text{ S(m/V)}^2 \rightarrow \gamma_{\text{gr}} = -8.9 \times 10^{23} \text{ 1/(Ws}^2\text{)}$$

$$n_{2,\text{Si}} = 2.5 \times 10^{-18} \text{ m}^2/\text{W} \rightarrow \gamma_{\text{Si}} = +4.7 \times 10^{22} \text{ 1/(Ws}^2\text{)}$$
- Kerr effect has a substantial impact on the transmission curves (though **SA** is the **dominant nonlinear effect**).
 \implies Appropriate pre-shifting of the operating wavelengths necessary to reach zero-transmission point.
- To cancel Kerr effect, an alternative self-focusing material with higher n_2 should be used \longrightarrow **Silicon Rich Nitride (SRN)**

$$n_{2,\text{SRN}} = 2.8 \times 10^{-17} \text{ m}^2/\text{W} \rightarrow \gamma_{\text{SRN}} = +2.5 \times 10^{23} \text{ 1/(Ws}^2\text{)}$$
 - Same order of magnitude of γ but still γ_{gr} dominates.
 - Kerr cancellation could be achieved by further device engineering.





To probe further

To probe further / Graphene Hot Electron Model (GHEM) (1/2)

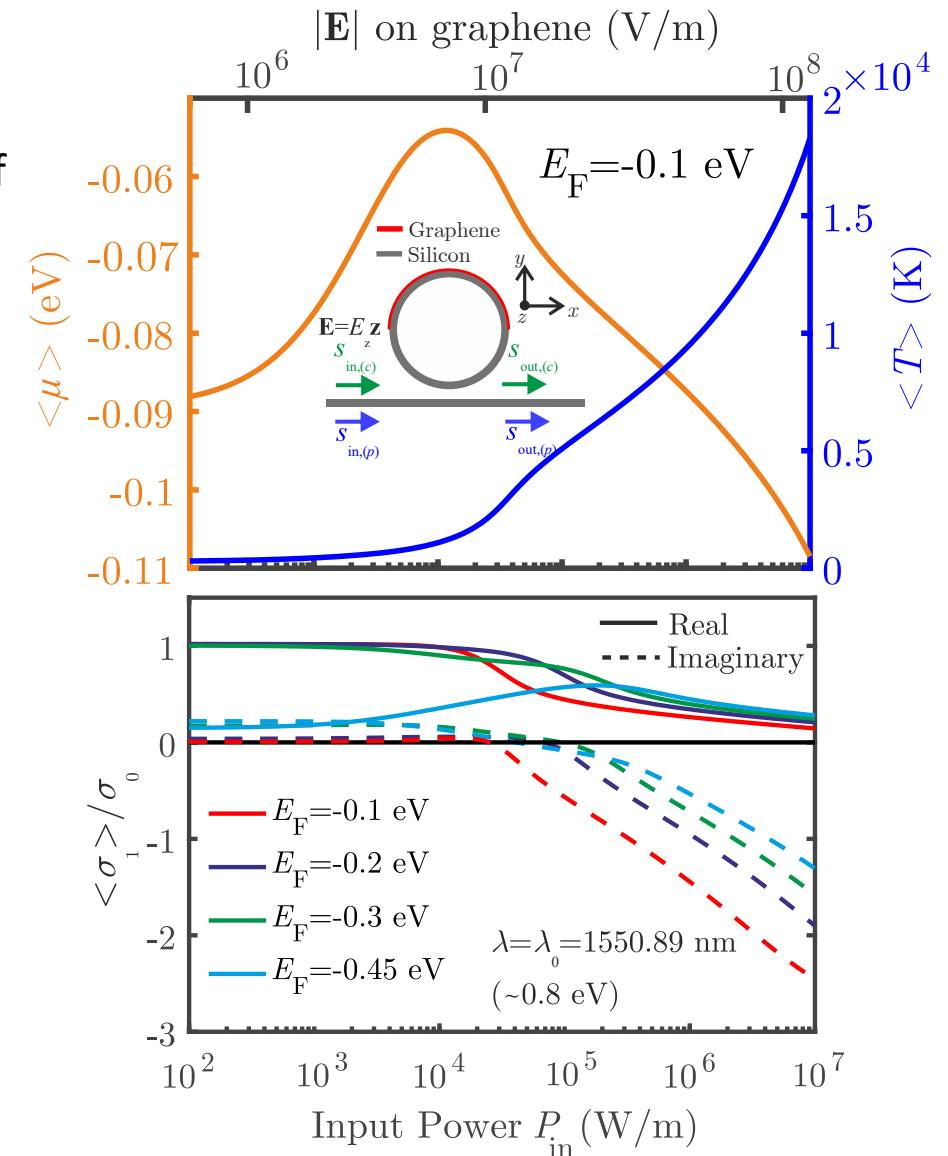
- **Thermodynamic analysis:** Under strong optical illumination, carriers are photo-generated and the plasma is heated resulting in variations of the chemical potential (μ) and temperature (T) → More precise description of graphene's nonlinearity.

- Subsequently, graphene surface conductivity is calculated by Kubo formulas as:

$$\sigma_1 = \sigma_1(\mu(\mathbf{E}_{||}), T(\mathbf{E}_{||}))$$

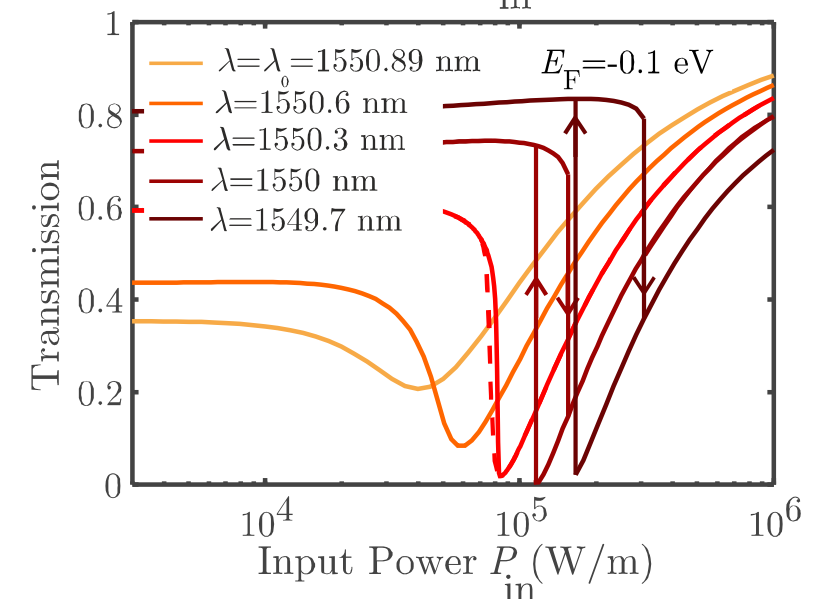
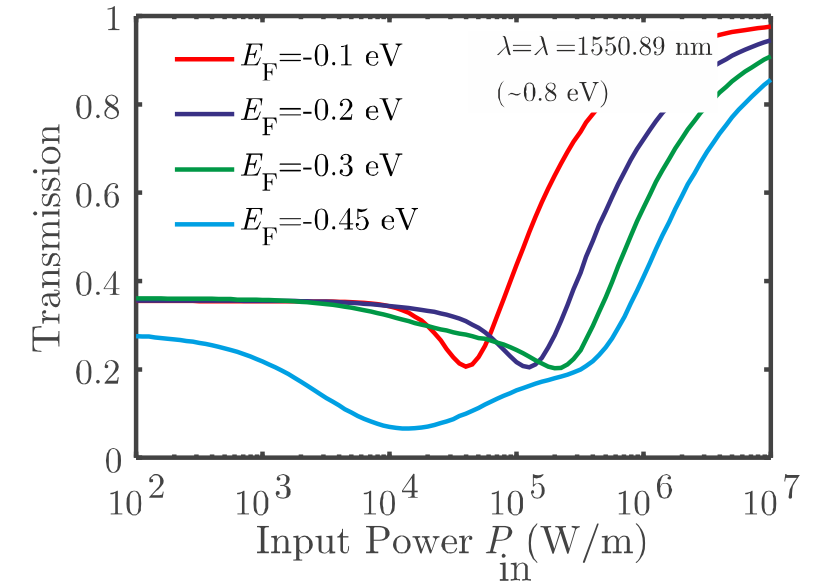
- For sufficiently low light intensity the thermalized chemical potential and temperature can be evaluated by energy-balance and electroneutrality conditions. [K. Alexander, ACS Photonics 2018, 5, 4944–4950](#)

- The model predicts:
 - SA or IA (Induced Absorption) depending on Fermi Energy (E_F).
 - **strong nonlinear refraction/phase effect.**



To probe further / Graphene Hot Electron Model (GHEM) (2/2)

- At critical coupling condition Transmission $\neq 0$ as a result of the strong nonlinear refraction effect.
 - Appropriate pre-shifting of the operating wavelength.
 - Appearance of **bistability** loops.
- **Outcome:**
 - Nonlinear refraction/phase effect is as strong as SA.
 - Stronger interplay between nonlinear refraction and SA/IA effects.
 - Similar qualitative behavior between GHEM and the diffusion model studied in this work.
- **Further consideration:** Is the graphene model valid for the intensity build-up conditions that apply to the resonators under consideration ?
- **Carrier diffusion in GHEM:**
 - $D = D(T)$ Ruzicka, **Opt. Mater. Express** 2(6), 708-716, 2012
 - Even more realistic and accurate modeling of graphene nonlinearity, though highly-nonlinear (D is power-dependent) and complicated.





Summary and Conclusions



Summary and Conclusions

□ Summary

- Model graphene SA by incorporating **finite relaxation time** and **carrier diffusion**.
- Demonstrate the qualitative and quantitative features of the model in a graphene-enhanced SOI resonator.
- Thoroughly study the effect of finite relaxation time and carrier diffusion both separately and collectively in CW and pulsed operation.

□ Conclusions

- **Finite relaxation time** affects:
 - directly the saturation intensity and as a result the input power requirement.
 - the trailing edges of the transmitted pulses in pulsed operation.
- **Carrier diffusion** results in:
 - higher input power requirement.
 - increased delay and distortion in pulsed operation.

□ Next steps

- Design the resonator on SRN/SiO₂ platform to cancel the Kerr effect.
- Apply appropriate GHEM in graphene-enhanced resonant configurations.



Thank you !

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The research work was supported by the Hellenic Foundation for Research and Innovation (H.F.R.I.) under the "First Call for H.F.R.I. Research Projects to support Faculty members and Researchers and the procurement of high-cost research equipment grant." (Project Number: HFRI-FM17-2086)